PERGAMON

International Journal of Multiphase Flow 27 (2001) 1301–1311

International Journal of Multiphase Flow

www.elsevier.com/locate/ijmulflow

Brief Communication

Interface shapes for two-phase laminar stratified flow in a circular pipe

T.S. Ng, C.J. Lawrence *, G.F. Hewitt

Department of Chemical Engineering & Chemical Technology, Imperial College of Science, Technology and Medicine, Prince Consort Road, London SW7 2BY, UK

Received 27 March 2000; received in revised form 30 December 2000

1. Introduction

In most pipeline codes for stratified two-phase flows, equivalent models or approximations are used in the computation of flow properties such as the phase fraction and the pressure gradient. If the interface is assumed to be smooth (which may be a reasonable approximation at low velocities) and the two fluids are assumed to be in laminar flow, analytical solutions are possible for cases where the geometry of the flow is simple. For more complex cases, accurate solutions for laminar stratified flows can be obtained using numerical methods.

The geometry of the system is shown in Fig. 1. The contact angle, θ, is the angle between the two phases and the pipe wall, measured through the denser fluid; the holdup, ε, is the fraction of the pipe cross-section occupied by the denser fluid. The contact angle and holdup values give a flat interface regardless of the Bond number if

$$\varepsilon = \frac{\theta - \cos\theta \sin\theta}{\pi}. \tag{1}$$

Gorelik and Brauner (1999) referred to this as the pseudo-gravity dominated case. If the holdup exceeds this value, i.e., at small contact angles or at large holdups, the shape of the interface is concave. Correspondingly, at small holdups or large contact angles, the interface shape is convex.

A flat interface is often assumed in models for stratified flow. Taitel and Dukler (1976) and Hall and Hewitt (1993) studied the flow properties of air–water systems assuming a flat interface and obtained reasonable results. When the density difference between the two fluids is large, the effect of surface tension and the contact angle at the pipe wall can generally be ignored. However, this assumption is carried into commercial codes that predict the behaviour of three-phase stratified

* Corresponding author. Tel.: +44-171 594 5622; fax: +44-171 594 5604.
E-mail address: c.lawrence@ic.ac.uk (C.J. Lawrence).

0301-9322/01/$ - see front matter
PII: S0301-9322(01)00005-2

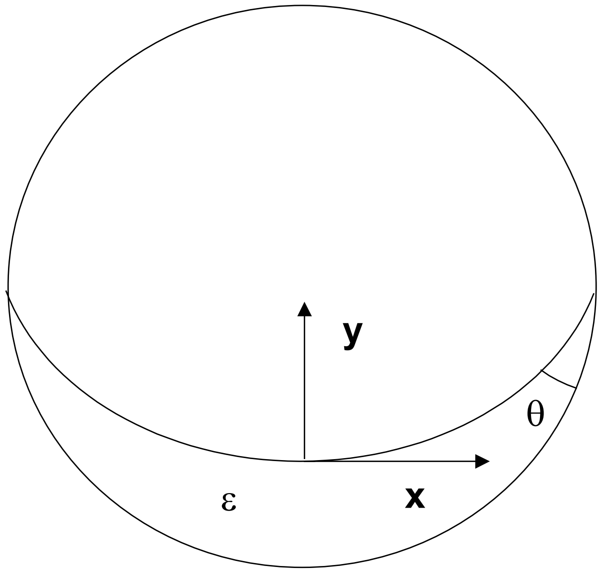

Fig. 1. The geometry and coordinate system.

flows, and the interface between the second (oil) phase and the third (water) phase is taken to be flat as well. For systems with small density differences, small diameters, large interfacial tensions, or in reduced gravity, interfacial curvature may be significant. The stratification due to the density difference is counteracted by the tendency of one fluid to wet the pipe's inner surface more than the other. These effects are characterised by the Bond number, B, given by

$$B = \frac{\Delta\rho \; g a^2}{\gamma_{AB}}, \tag{2}$$

where $\Delta\rho$ is the density difference between the two fluids, g the gravitational constant, a the radius of the pipe and γ_{AB} is the interfacial tension. The larger the Bond number, the more closely the interface approaches a flat surface.

Flows with interfaces defined by a circular arc or an eccentric circle have also been modelled both analytically and numerically by Bentwich's (1976), Brauner et al. (1996) and Soleimani (1999). However, most of these analyses do not take into account the full effects of surface tension and the contact angle between the fluids and the wall. In Bentwich's (1976) analytical solution, the underlying principle is that the masses in the pipe would be distributed in such a way that the potential energy of the system is a minimum under the influence of gravity, interfacial tension and wetting effects. Brauner et al. (1996) also considered the minimisation of the energy of the fluid–fluid system with the interface approximated by a circular arc. The solution is presented as an interface monogram. Soleimani (1999) determined the interfacial shape of a liquid–liquid system by solving the Young–Laplace equation numerically. He concluded that the shape of the interface depended on the contact angle, the Bond number and the ratio of in situ volume fraction of the two fluids.

There remains a need to predict the shape of the interface accurately in an efficient and robust way. If this is not achieved, valuable information regarding the flow properties might be lost when some physical properties, usually the interfacial tension and the contact angle of the two-fluid system, are not fully considered. Two groups of workers (Gorelik, 1999; Gorelik and Brauner, 1999; Ng et al., 1999a,b) have independently obtained an exact analytical solution for the interface shape by solving the Young–Laplace equation with contact angle boundary condition. Gorelik and Brauner (1999) recently published the solution, along with extensive analysis of the differences between the exact solution and the approximate, circular arc solution used by Brauner et al. (1996). They concluded that the circular arc approximation provides a very good model for all values of the dimensionless parameters.

In this work, a catalogue of the different exact interface shapes for various Bond numbers, contact angles and holdups is presented. A companion paper will address the calculation of the flow fields for this system.

2. Mathematical formulation

The exact analytical solution for the location of the interface is identical to the one proposed by Gorelik and Brauner (1999). The approach taken here is the simultaneous solution of the Young–Laplace equation with appropriate boundary conditions and a constraint on the holdup of the denser fluid. The equations governing the problem and its solution are given below, but further details can be found in Ng et al. (1999a,b) or Gorelik and Brauner (1999). The formulation is dimensionless, and the pipe is of unit radius.

The Young–Laplace equation can be written as a pair of differential equations involving the interface position and curvature

$$\frac{\mathrm{d}X}{\mathrm{d}\phi} = \frac{b\cos\phi}{1+Y}, \tag{3}$$

$$\frac{\mathrm{d}Y}{\mathrm{d}\phi} = \frac{b\sin\phi}{1+Y}, \tag{4}$$

Eqs. (3) and (4) are solved to give the position of the fluid–fluid interface in terms of the scaled horizontal and vertical coordinates,

$$X = \pm\left[-E\left(\frac{\phi}{2}, -4b\right) + (1+2b)F\left(\frac{\phi}{2}, -4b\right)\right] \tag{5}$$

and

$$Y = \pm\left[-1 + \sqrt{1+2b-2b\cos\phi}\right], \tag{6}$$

where ϕ is a parameter of the interface which is defined as the angle between the tangent to the interface and the horizontal in the positive X direction. F represents the elliptic integral of the first kind, given by

$$F\left(\frac{\phi}{2}, -4b\right) = \int_0^{\phi/2} \left[1 + 4b \sin^2 t\right]^{-1/2} \mathrm{d}t \tag{7}$$

and E is the elliptic integral of the second kind,

$$E\left(\frac{\phi}{2}, -4b\right) = \int_0^{\phi/2} \left[1 + 4b \sin^2 t\right]^{1/2} \mathrm{d}t. \tag{8}$$

At this stage, the solution is characterised by a single parameter b, which is a modified Bond number, based on the interface curvature, given by

$$b = \frac{B}{\kappa_0^2} \tag{9}$$

with κ_0 being the reference curvature at the centre of the interface. The complete family of solutions is shown in Fig. 2. Here, the full potential range of ϕ is shown from $\phi = -\pi$ to $+\pi$. Any interface shape may be obtained by taking a (symmetric) part of one of the arcs in Fig. 2 and embedding it in a circle of suitable size representing the pipe.

The solution given by Eqs. (5) and (6) is re-scaled to physical coordinates in the form

$$(x, y) = \frac{(X, Y)}{\sqrt{Bb}} \tag{10}$$

and the range of ϕ is adjusted to satisfy the boundary condition at the contact line, which requires that

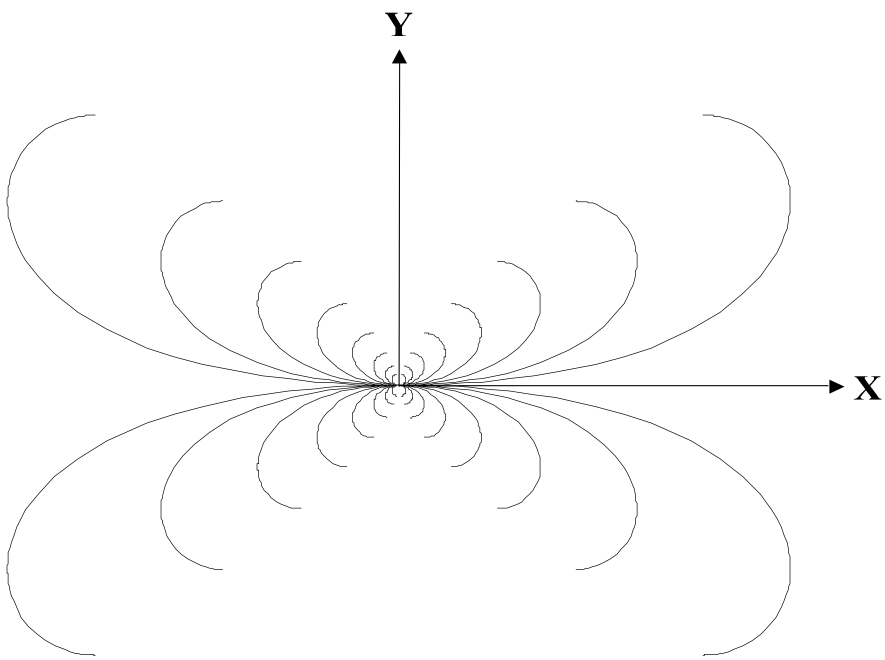

Fig. 2. Family of curves for interface shape, given by Eqs. (3) and (4) with $b = \pm 0.25$, ± 0.5, ± 1, ± 2, ± 4, ± 8, ± 16, ± 32. The shape changes from circular for small b to oblate for large b.

$$(Bb)^{1/2}\sin(\phi_{\max}+\theta) = (1+2\mathrm{b})F\left(\frac{\phi_{\max}}{2},-4b\right) - E\left(\frac{\phi_{\max}}{2},-4b\right), \tag{11}$$

where $\phi_{\max}$ is measured at the contact point of the interface and the wall.

The holdup of the denser fluid, ε, is determined from the areas of segments and sectors within the cross-section of the pipe. The resulting expression applies for both concave and convex interfaces:

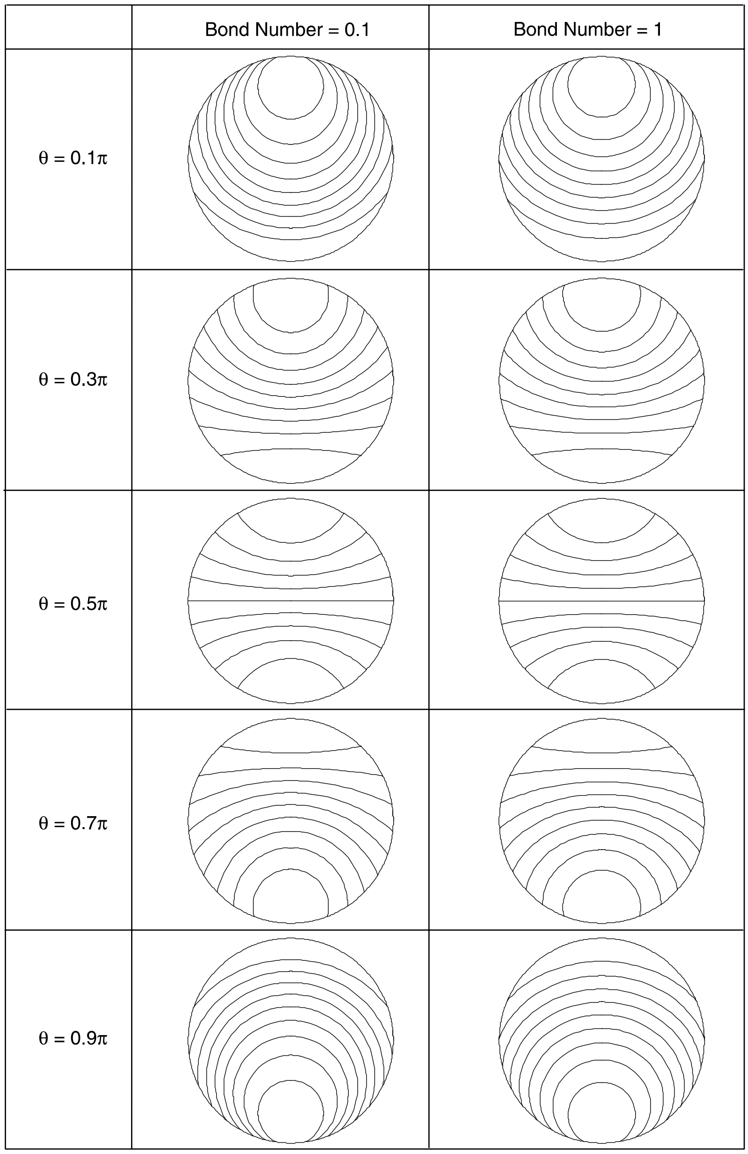

Fig. 3. Variation of the interface shape with holdup for constant Bond number and contact angle. $\varepsilon = 0.1, 0.2, \ldots, 0.9$.

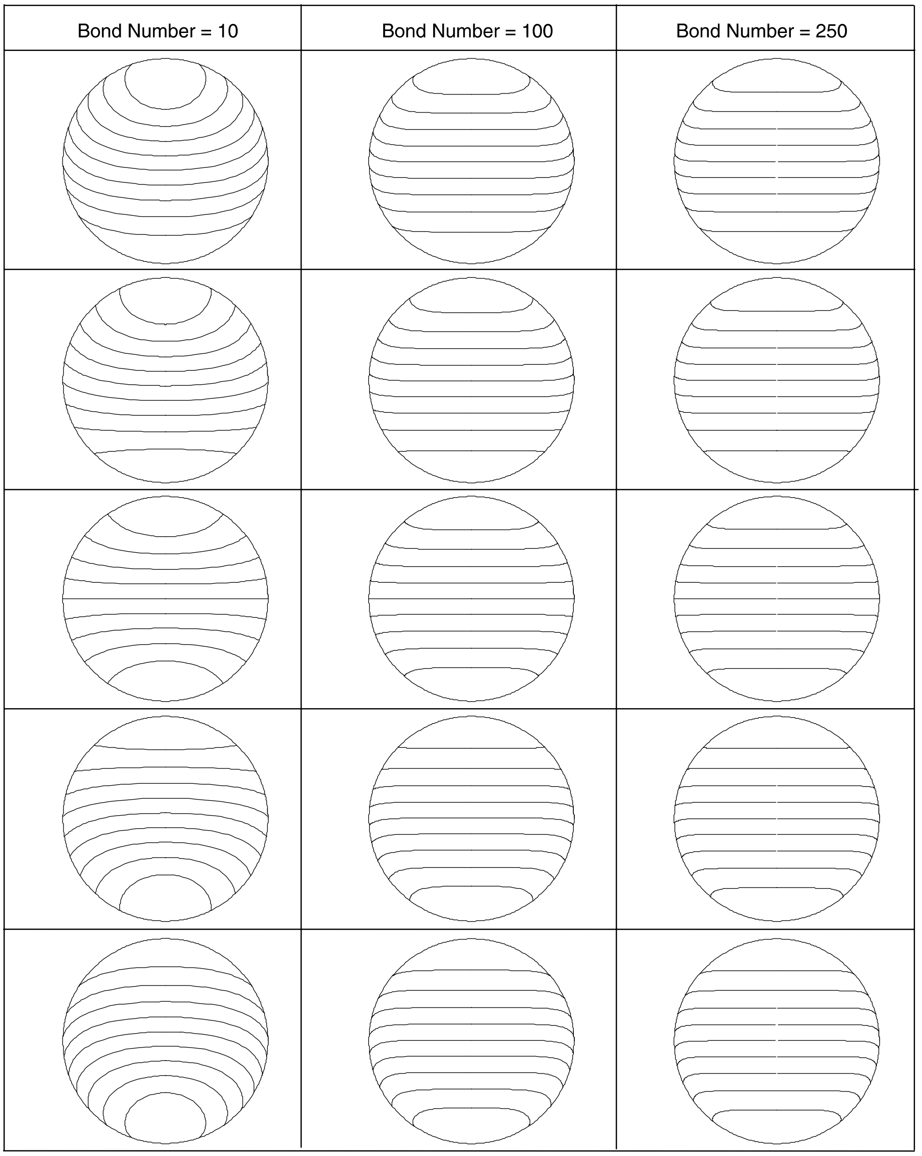

Fig. 3. (*continued*)

$$\varepsilon = \frac{\phi_{\max} + \theta}{\pi} - \frac{1}{2\pi}\sin 2(\phi_{\max} + \theta) + \frac{2}{B\pi}\sin\phi_{\max} - \frac{2}{\pi\sqrt{Bb}}\sin(\phi_{\max} + \theta)\sqrt{1 + 2b - 2b\cos\phi_{\max}}, \tag{12}$$

where Eq. (11) has been used to simplify Eq. (12).

Eqs. (11) and (12) are solved simultaneously to obtain the values of $\phi_{\max}$ and b, for given values of θ, B and ε. Accounting for slight simplification of the notation, the solution is identical to that presented by Gorelik and Brauner (1999).

3. Results

The full catalogue of exact interface shapes is shown in three different ways in Figs. 3–5. Fig. 3 shows the interface shapes for different holdups, from 0.1 to 0.9 increasing by 0.1, at a constant contact angle and Bond number. Fig. 4 shows the effect of varying the contact angle between the two fluids and the wall from 0.1π to 0.9π in steps of 0.1π, keeping the holdup and Bond number

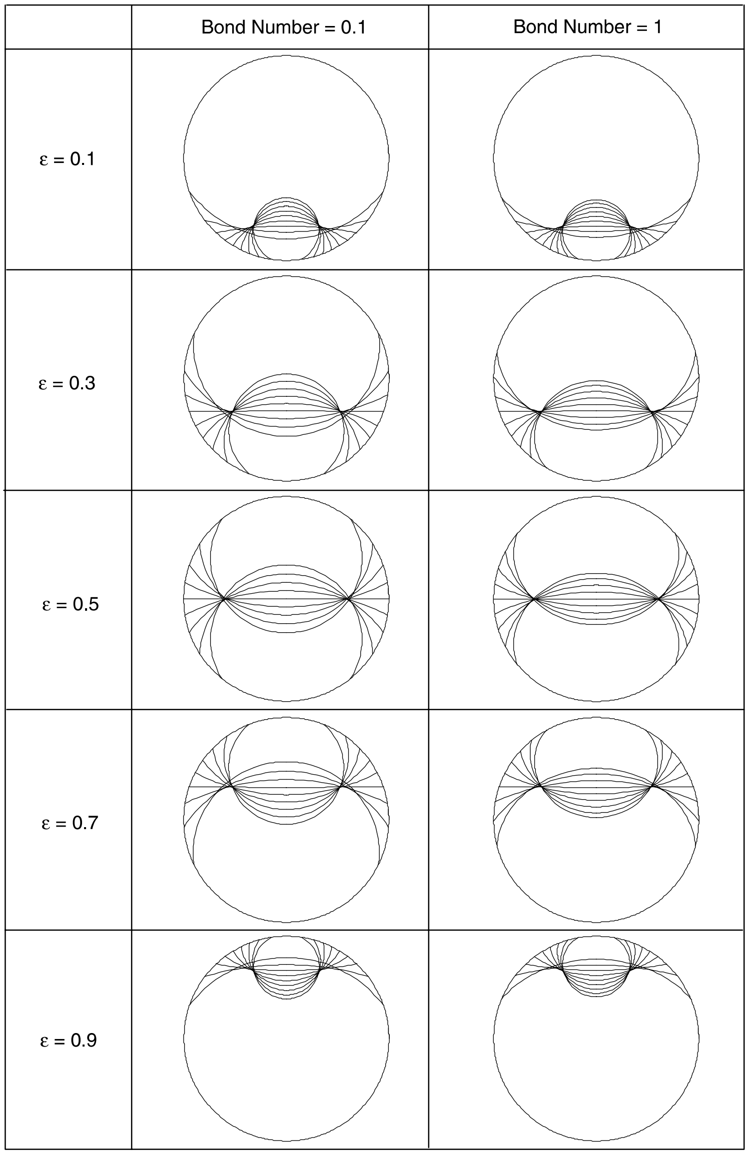

Fig. 4. Variation of the interface shape with contact angle for constant Bond number and holdup. $\theta = 0.1\pi$, $0.2\pi, \ldots, 0.9\pi$.

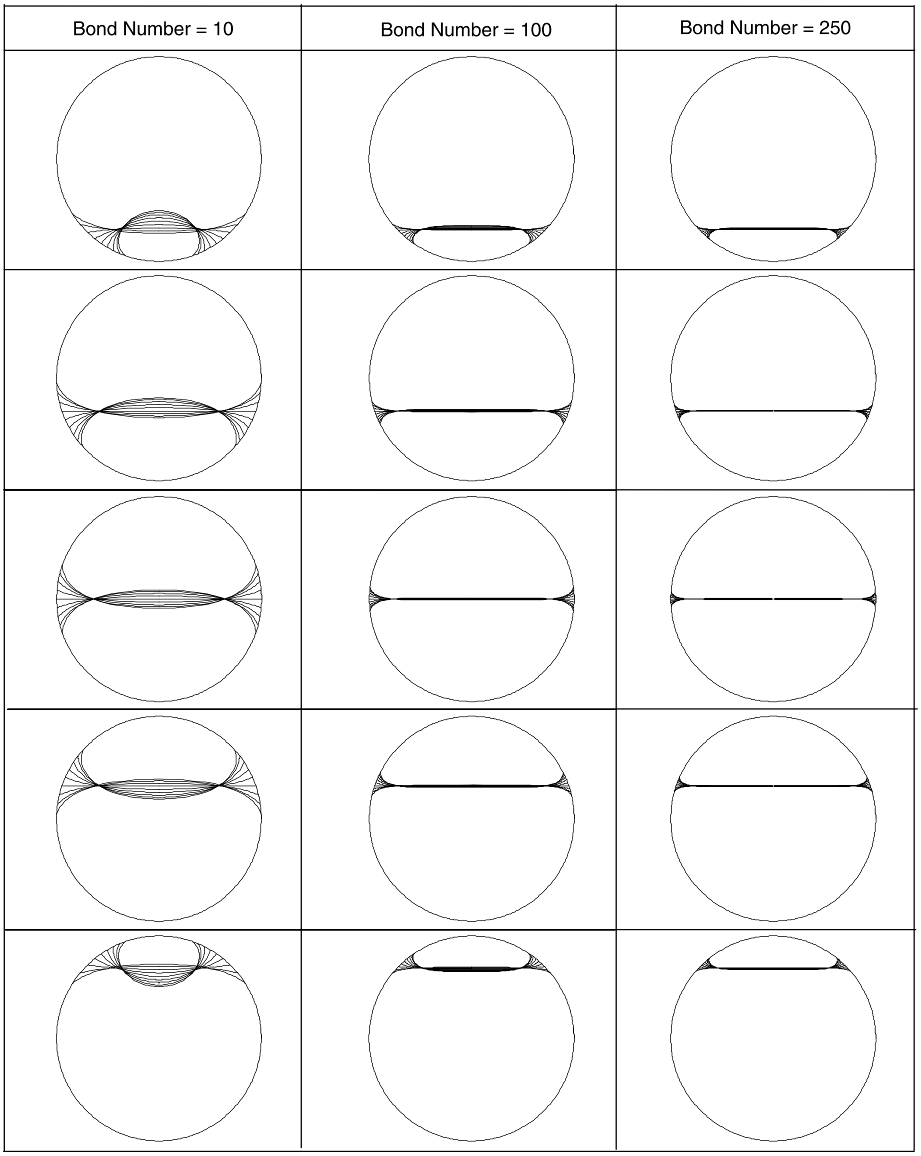

Fig. 4. (*continued*)

constant. Finally, Fig. 5 gives the fluid–fluid interfaces for fixed holdups and contact angles for increasing Bond numbers from 0 to 180 in steps of 20.

For small Bond numbers, surface tension is dominant and the interface approaches a circular arc shape. When the Bond number increases, the interface shape changes from a circular one to an approximately elliptical one. At large Bond numbers, the interface is almost flat with a slight meniscus at the ends near the walls of the pipe; the difference in densities of the two fluids outweighs the effect of surface tension. In addition, the difference in height of the interface between the centre and the ends (at the walls) of the interface decreases with increasing Bond number.

For a given Bond number, the interface shape is symmetrical with respect to complementary changes of the holdup and contact angle ($\varepsilon \rightarrow 1-\varepsilon,\ \theta \rightarrow \pi-\theta$). For example, the shape for a holdup of 0.1 and a contact angle of 0.1π is the mirror image of the one for a holdup of 0.9 and a contact angle of 0.9π. The interface shape is highly concave at small contact angles, but becomes convex when the contact angle increases.

When the Bond number is small, at low holdups and large contact angles, the denser phase curls up into an eccentric circular core. Near this limit, the flow configuration is between those of

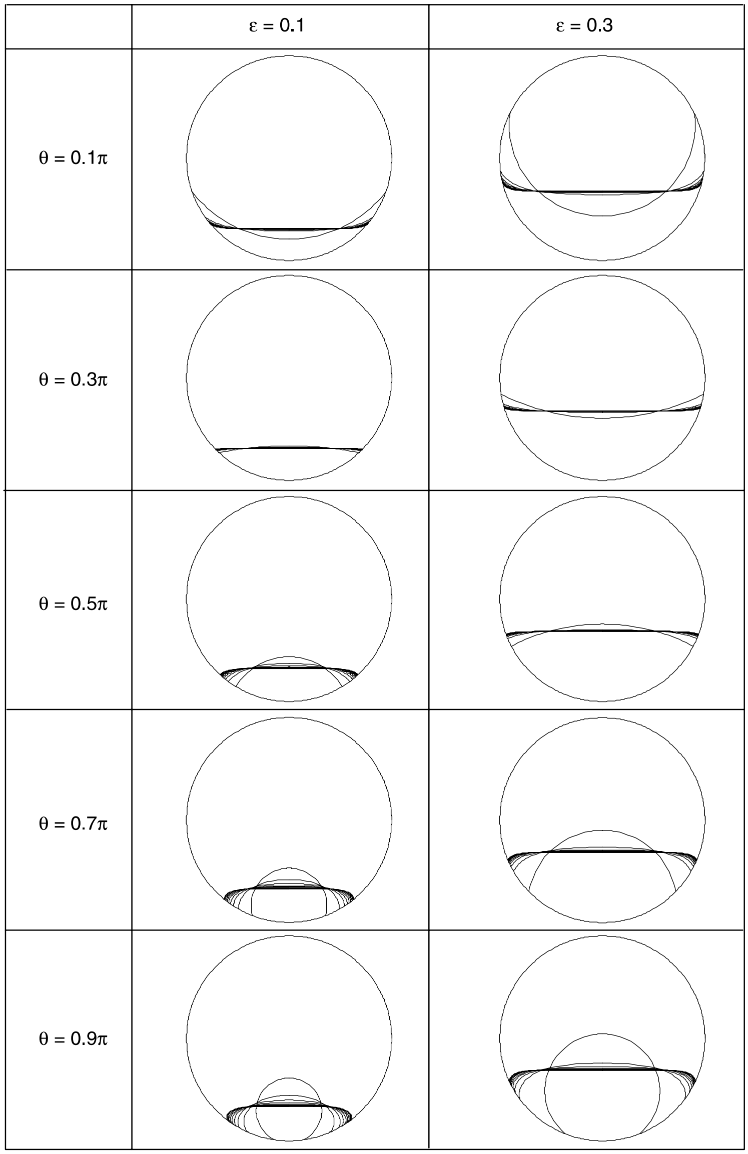

Fig. 5. Variation of the interface shape with Bond number for constant holdup and contact angle. $B = 0, 20, \ldots, 180$.

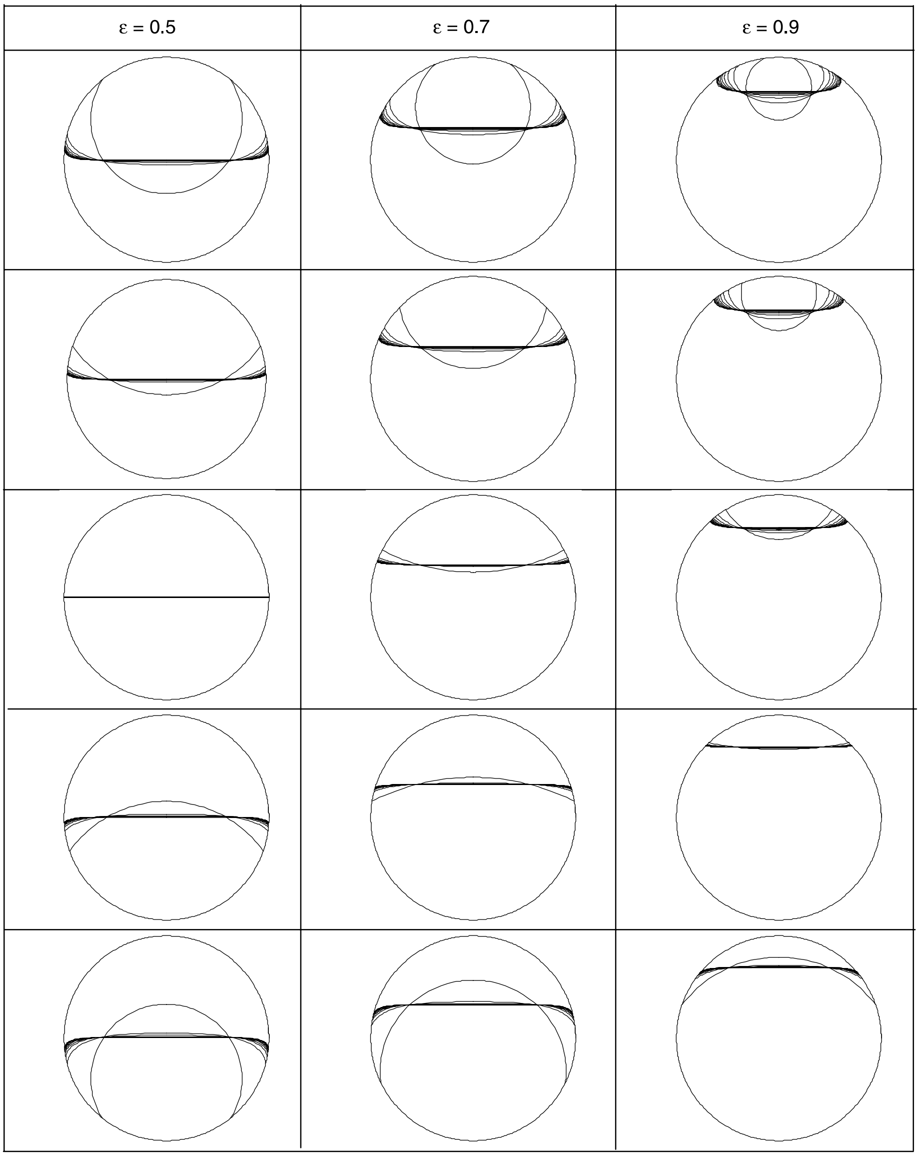

Fig. 5. (*continued*)

core-annular flow and stratified flow. On the other end of the range, at high holdups, the less dense phase becomes an eccentric core when the contact angle is small.

4. Conclusion

The possible interface shapes are portions of a family of curves described by a single parameter b, which is given by B/κ_0^2. The appropriate value of b (hence κ_0) is determined as part of the solution along with the portion of the curve to use. The solution is important in the prediction of system performance and flow characteristics including the limiting conditions of both annular and stratified flows.

Acknowledgements

This work was supported by the WASP consortium. We would like to thank Professor Costas Pozrikidis for helpful discussion, and Professor Neima Brauner for her encouragement to publish this work.

References

Bentwich, M., 1976. Two phase axial laminar flow in a pipe with naturally curved interface. Chem. Eng. Sci. 31, 71–76.

Brauner, N., Rovinsky, J., Moalem Maron, D., 1996. Determination of interface curvature in stratified two phase systems by energy considerations. Int. J. Multiphase Flow 22, 1167–1185.

Gorelik, D., 1999. Interface shape between two immiscible layers and its effect on the flow characteristics. M.Sc. Thesis, Faculty of Engineering, Tel Aviv University.

Gorelik, D., Brauner, N., 1999. The interface configuration in two-phase stratified pipe flows. Int. J. Multiphase Flow 25, 977–1007.

Hall, A.R.W., Hewitt, G.F., 1993. Application of two-fluid analysis to laminar stratified oil-water flows. Int. J. Multiphase Flow 19, 711–717.

Ng, T.S., Lawrence, C.J., Hewitt, G.F., 1999a. Exact interface shape calculations for a two-phase stratified flow system. Multiphase flow systems report WASP/46, MPS/122. Imperial College, London.

Ng, T.S., Lawrence, C.J., Hewitt, G.F., 1999b. A detailed analysis of smooth stratified laminar flow in a pipe. In: Proceedings of the 37th European Two-Phase Flow Group Meeting, London, 28–30 September 1999.

Soleimani, A., 1999. Phase distribution and associated phenomena in oil–water flows in horizontal tubes. Ph.D. Thesis, University of London.

Taitel, Y., Dukler, A.E., 1976. A model for predicting flow regime transitions in horizontal and near horizontal gas–liquid flow. AIChE J. 22, 47–55.